Partially Composite Sandwich Panel Deflections

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Abstract: A continuum model for evaluating partially composite insulated concrete sandwich panel deflections with discrete shear connectors due to differential volume changes such as those caused by differences in temperature between the wythes is evaluated for accuracy by comparison with finite-element analysis. Design equations for computing these deflections are provided. Analysis of sample panels indicates that the continuum model provides acceptable accuracy for calculating thermal bowing of wall panels. The resulting equations show that, for long panels, the amount of thermal bowing in the panels is relatively insensitive to the stiffness of the connectors. This implies that long insulated sandwich panels with low connecting-layer stiffness will experience nearly the same amount of thermal bowing as fully composite panels.

INTRODUCTION

Insulated sandwich panels are widely used to provide a structural shell for buildings. These panels typically consist of two outside layers (wythes) surrounding an insulation layer. The outer layers are usually constructed of precast or prestressed concrete and are connected through the insulation layer to form a structurally composite panel. This composite action causes the panel to deflect or bow when the structural wythes experience differences in temperature or humidity due to the presence of the insulation wythe.

A common method of connecting the concrete wythes uses truss-shaped connectors to provide shear transfer between the wythes (Fig. 1). These connectors are embedded into each concrete wythe, anchored to longitudinal wythe reinforcement and transfer shear primarily through tension and compression forces in the connector diagonals. Steel trusses are commonly used and provide sufficient shear stiffness to treat the panel as fully composite. However, the conductivity of steel causes loss of insulative value, and the high stiffness increases differential temperature deflections. Connectors made of newer materials provide better insulation and reduce panel composite action.

This paper provides design equations for computing deflections due to differential volume changes for partially composite panels with truss-type connectors. These equations are compared with finite-element analysis to assess their accuracy, and are used to investigate the relationship between composite action and thermal bowing.

DIFFERENTIAL EQUATIONS

Computing deflections for partially composite sandwich panels requires accounting for both the bending deformation of the concrete wythes and the relative shearing deflection between the wythes through the connecting layer. Several equivalent derivations of the basic equations are given by Allen (1969), Holmberg and Plem (1986), Gordaninejad and Bert (1989), Frostig and Baruch (1990), Paydar and Park (1990), and Ha (1992). The following derivation follows that given by Holmberg and Plem (1986).

Fig. 2 shows the deformation of a differential panel element. This deformation consists of two components: (1) that due to panel curvature, $zd\theta$; and (2) an offset due to the shearing deformation between the wythes, $q_{,x}dx/2$. For small displacements, the panel curvature is $d\theta = y_{,xx}dx$. Summing the moment due to each deformation component and rearranging yields the following:

$$y_{,xx} = \frac{M}{EI} + \frac{\alpha^2}{2r} q_{,x} \tag{1}$$

where $\alpha^2 = (I - 2I_w)/I$; x = distance along the length of the panel; y = upward displacement of the panel; q = relative shearing displacement between the centroids of the top and bottom wythes; b = width of the panel; M = applied moment; E = wythe modulus of elasticity; I_w = moment of inertia of each wythe; and I = moment of inertia of the entire panel cross section.

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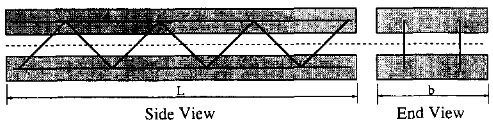

FIG. 1. Insulated Sandwich Panel with Truss-Type Connectors

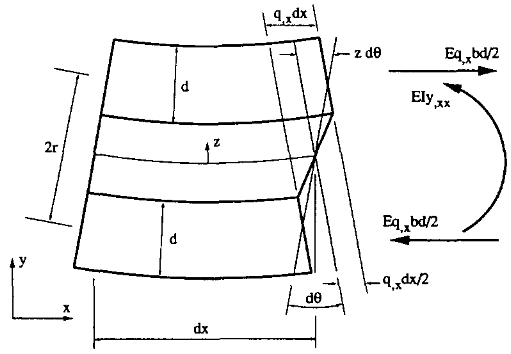

FIG. 2. Differential Panel Element

The shear stress in the connecting layer is taken as Kq, where K = shear stiffness of connecting layer. This shear stress is a superposition of the stress due to panel curvature and the stress due to the shear displacement offset between the wythes. The former is given by the standard shear-stress formula and the latter is equal to the change in wythe force $Eq_{,x}bd/2$ over the length dx of $Edq_{,xx}dx/2$. Equilibrium of the shear forces in the connecting layer requires the following:

$$Edry_{,xxx} - \frac{1}{2}Edq_{,xx} + Kq = 0 \tag{2}$$

Substituting the derivative of (1), multiplying by $2/Ed$, and rearranging yields the following:

$$\beta^2 q_{,xx} - \chi^2 q = 2r\frac{M_{,x}}{EI} \tag{3}$$

where $\beta^2 = (1 - \alpha^2)$ and $\chi^2 = 2K/Ed$. Given the moment on the section, (3) can be solved for q. Given q, (1) can be integrated twice to give the deflection.

Eqs. (1) and (3) assume negligible contribution of the connecting layer to the resisting moment on the section and a homogeneous connecting layer with a shear modulus of K. For concrete sandwich panels with truss connectors, the first assumption is generally valid. The second assumption causes local errors in the distribution of q along the length of the panel. However, if K is computed as the force per unit length required to cause a unit shearing deformation between the wythes, the displacement error should be small provided enough truss members are used. This is verified as follows by comparison with finite-element analysis using discrete connectors.

BOUNDARY CONDITIONS

Boundary conditions on q and y at the panel ends, $x = x_0$, and $x = x_L$ are given for three cases.

Case 1. Simple support: Because each wythe is free to rotate, the moment at the ends of each wythe is zero. Also, the overall panel moment is zero. Therefore, from (1), we get the following:

$$y_{,xx}(x_0) = y_{,xx}(x_L) = 0, \text{ and } q_{,x}(x_0) = q_{,x}(x_L) = 0 \tag{4a,b}$$

Case 2. Fixed support (no wythe rotation): Because relative displacement between the wythes is prevented in this case, then

$$y_{,x}(x_0) = y_{,x}(x_L) = 0, \text{ and } q(x_0) = q(x_L) = 0 \tag{5a,b}$$

Case 3. Fixed support (wythe rotation allowed): In this case, relative displacement between the wythes is prevented, but wythe rotation is free. The moment in each wythe is therefore zero, with the total panel moment taken by the relative displacement restraint. Therefore, from (1), we get the following:

$$y_{,xx}(x_0) = y_{,xx}(x_L) = 0, \; q_{,x}(x_0) = -\frac{2rM(x_0)}{\alpha^2 EI}, \text{ and } q_{,x}(x_L) = -\frac{2rM(x_L)}{\alpha^2 EI} \tag{6a–c}$$

CONNECTOR STIFFNESS

Eqs. (1) and (3) require evaluating the stiffness of the connecting layer. This stiffness is computed in an average sense over a unit length of panel. Because the connectors are embedded into the structural wythe, several end-restraint conditions for the connectors are possible depending upon the connector support provided by the wythes.

Three restraint conditions are shown in Fig. 3. In Fig. 3(a) (condition 1), the truss members are pinned at the center of each wythe. In Fig. 3(b) (condition 2), the concrete embedment

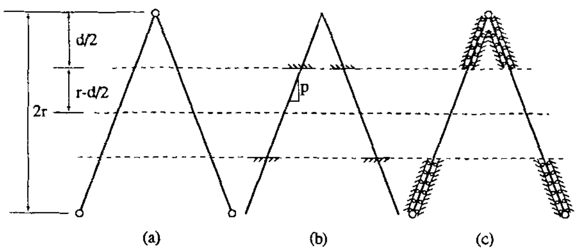

FIG. 3. Connector-Embedment Types: (a) Pinned at Wythe Center; (b) Fixed at Wythe Embedment; (c) Laterally Supported within Wythe

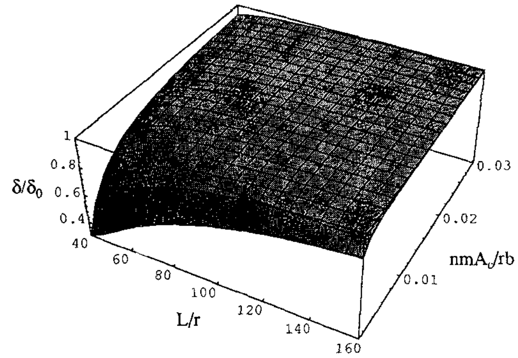

FIG. 4. δ/δ_0 for d/r = 1

provides full lateral and axial fixity at the point of embedment. In Fig. 3(c) (condition 3), slip can occur along the embedment length, and the wythe provides lateral restraint along the embedment length with axial fixity at the center of each wythe.

The connecting layer stiffness K is computed for each of the foregoing three conditions as the force per unit area parallel to the wythes required to sustain a unit relative displacement between the wythes. For the connectors shown in Figs. 1 and 3, the tributary area is $2rb/mp$, where m = number of connector rows across the panel width, b; and p = connector slope.

Condition 1: Truss Action Only

In this case, connectors of area A_c and modulus of elasticity E_c are pinned at the center of each wythe. For a unit relative shear deformation between the wythes, the elongation of the connector, e, is as follows:

$$e = \frac{1}{\sqrt{1 + p^2}} \tag{7}$$

Multiplying by the connector stiffness gives the axial connector force, F, as follows:

$$F = \frac{E_c A_c p}{2r(1 + p^2)} \tag{8}$$

The shear stiffness is the component of F parallel to the wythes, divided by its tributary panel area, as follows:

$$K = \frac{A_c E_c p^2 m}{4r^2 b(1 + p^2)^{3/2}} \tag{9}$$

Condition 2: Full Embedment Fixity

Assuming the concrete embedment provides complete fixity of the connector at the point of connector embedment, the shear stiffness is computed similarly to Fig. 3(a). However, the length of the connector is taken from face of wythe to face of wythe, and the connector's bending stiffness is included in the force. The shear stiffness is as follows:

$$K = \frac{E_c p^2 m}{2rb(1 + p^2)^{5/2}(2r - d)^3}[(1 + p^2)(2r - d)^2 A_c + 12p^4 I_c] \tag{10}$$

where I_c = moment of inertia of each connector.

Condition 3: Lateral Embedment Restraint

Here, the axial connector length extends from wythe center to wythe center, and the bending length extends from wythe face to wythe face. The shear stiffness is again computed similarly as follows:

$$K = \frac{E_c p^2 m}{4rb(1 + p^2)^{3/2}}\left[\frac{A_c}{r} + \frac{24p^4 I_c}{(1 + p^2)(2r - d)^3}\right] \tag{11}$$

To compare the three cases, take r = 80 mm, d = 80 mm, p = 1, and compute A_c and I_c for a bar of diameter d_b = 11.3 mm. For Fig. 3(a): $K = 1.38 \times 10^{-3}$ $(E_c m/b)$; for Fig. 3(b): $K = 2.78 \times 10^{-3}$ $(E_c m/b)$; and for Fig. 3(c): $K = 1.40 \times 10^{-3}$ $(E_c m/b)$.

It is evident that the bending contribution to the connector stiffness is small. However, if the embedment provides axial restraint to the connectors (condition 2), the increase in stiffness is significant. Preliminary test results for fiber-reinforced plastic (FRP) connectors indicate that slip occurs along the embedment length justifying the use of either condition 1 or 2 (Einea 1992; Einea et al. 1994). Because of the relative simplicity of condition 1 over 3 and the small stiffness provided by lateral restraint of the connectors, condition 1 is used in the following for FRP connectors.

UNIFORM MOMENT

The displacement caused by a uniform temperature differential between the two wythes of the panel is the same as that caused by the application of equal and opposite end moments, M_T, to the sandwich panel. By superposition, the magnitude of M_T is opposite to the moment of the force required to compress the expanded (top) wythe back to its original length, $bdE\alpha_T\Delta T$ (Ghali and Nevile 1989). Thus

$$M_T = -rbdE\alpha_T\Delta T \tag{12}$$

where α_T = wythe coefficient of thermal expansion; and ΔT = difference in temperature between the top and bottom wythe. For other types of differential volume change, $\alpha_T\Delta T$ may be replaced with the appropriate strain difference between the two wythes.

The solution to (1) and (3) for $M = M_T$ and simple supports [(4)] is the following:

$$q(x) = -\frac{2M_T}{EI}\frac{r\beta}{\alpha^2\chi}\frac{\sinh(\chi/\beta x)}{\cosh(\chi L/2\beta)};\quad y(x) = \frac{M_TL^2}{8EI}\left\{2\left(\frac{2\beta}{\chi L}\right)^2\left[1 - \frac{\cosh(\chi/\beta x)}{\cosh(\chi L/2\beta)}\right] + 4\left(\frac{x}{L}\right)^2 - 1\right\} \tag{13, 14}$$

where L = overall length; and x = distance from the center line of the panel.

The maximum deflection, δ, occurs at the center of the panel ($x = 0$), as follows:

$$\delta = -\frac{M_TL^2}{8EI}\left[1 - \frac{2}{\psi^2}(1 - \operatorname{sech}\psi)\right] = \delta_0\left[1 - \frac{2}{\psi^2}(1 - \operatorname{sech}\psi)\right] \tag{15}$$

where

$$\delta_0 = -\frac{M_TL^2}{8EI};\quad \text{and}\quad \psi = \frac{\chi L}{2\beta} \tag{16, 17}$$

COMPUTATION OF DISPLACEMENT FACTOR

The displacement factor, δ/δ_0, for a simply supported panel assuming connector-embedment condition 1 and a connector slope of unity is computed as follows:

$$\beta^2 = \frac{1}{1 + 12(r/d)^2};\quad \text{and}\quad \chi^2 = \frac{2K}{Ed} = \frac{A_c m}{4\sqrt{2}r^2 b}\frac{E_c}{E}\frac{1}{d} = \frac{1}{4\sqrt{2}}\frac{A_c mn}{rb}\frac{1}{rd} \tag{18, 19}$$

Substituting the foregoing definitions of χ and β into ψ gives the following:

$$\psi^2 = \frac{1}{16\sqrt{2}}\left(\frac{L}{r}\right)^2\left(\frac{A_c nm}{rb}\right)\frac{r}{d}[1 + 12(r/d)^2] \tag{20}$$

Fig. 4 shows the variation of δ/δ_0 for $d/r = 1$ as a function of nmA_c/rb and L/r. As the L/r increases, the effect of the connector stiffness on the deflection of the panel diminishes. This indicates that for longer sandwich panels, a small connecting-layer stiffness can cause thermal bowing nearly equal to that for a panel with a large connecting-layer stiffness.

NUMERICAL EXAMPLES

In the following examples, midspan panel displacements are computed for two different panels using (15)–(20). These displacements are compared with finite-element analysis to verify the use of the continuum model. Table 1 shows the geometric and material data for both examples.

Example 1. Long panel: Consider a long sandwich panel with a high L/r ratio but low connector stiffness (Table 1), subject to a temperature difference of 25°C between wythes causing equivalent end moments, M_T [(12)] of −146 kN·m. The deflection of a fully composite panel in this case [(16)] is δ_0 = 21.1 mm. From (15)–(20), the value of δ/δ_0 is 0.823, giving a midspan deflection of 17.4 mm. Fig. 5 shows the two-dimensional finite-element mesh for this example. Each wythe is modeled from support to center line using 260 four-node QM6 elements. The top wythe is subject to a uniform temperature increase of 25°C. Symmetry boundary conditions are applied at the center line with support at the other end as shown. The connectors are modeled using two-node truss elements pinned to the center of each wythe, simulating connector-restraint condition 1. The average midspan displacement of the finite-element model is 17.3 mm. The 0.5% displacement error is caused by the difference between the discrete connector model used in the finite-element analysis and the continuum model.

TABLE 1. Example Data

Quantity (1)	Unit (2)	Example 1 values (3)	Example 2 values (4)
width, b	m	3	1.22
length, L	m	10	2.44
wythe thickness, d	mm	80	76.2
insulation thickness	mm	80	76.2
I	mm^4	3.33×10^9	1.17×10^9
r	mm	80	76.2
E_w	MPa	26,000	24,900
E_c	MPa	24,000	20,700
α_t	mm/mm°C	1.17×10^{-5}	1.17×10^{-5}
d_b	mm	11.3	9.53
A_c	mm^2	100	71.3
m	—	3 connectors	3 connectors
nmA_c/rb	—	0.00115	0.0019

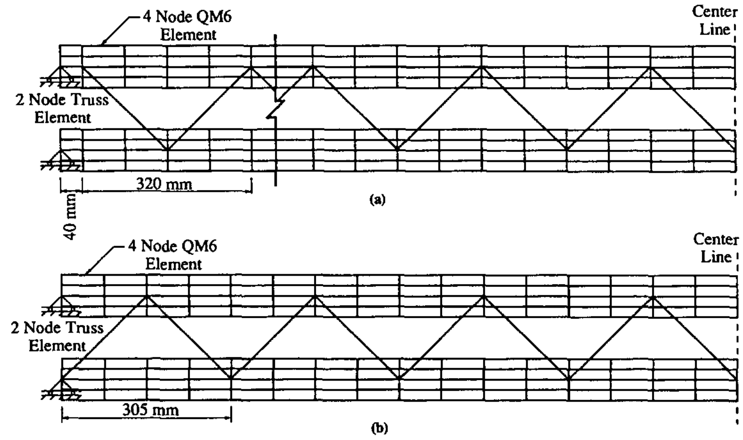

FIG. 5. Example Finite-Element Meshes: (a) Long Panel; (b) Short Panel

Example 2. Short panel: Consider a similar analysis of a short panel (Table 1). The equivalent moment in this case [(12)] is -51.5 kN·m. The deflection of the fully composite panel [(16)] is $\delta_0 = 1.32$ mm. From (15)–(20), the value of δ/δ_0 is 0.32, giving a midspan deflection of 0.424 mm. The finite-element model for this example is shown at the bottom of Fig. 5. Sixty-four four-node QM6 elements are used to model each wythe with the same load, boundary conditions, and connectors as is example 1. The average midspan displacement from finite-element analysis is 0.419 mm, giving an error of 1.2%.

CONCLUSION

Computating panel deflections for partially composite insulated sandwich panels requires considering the effect of relative shear displacement between wythes. Eqs. (15)–(20) give the deflection due to thermal bowing of panels using truss-type connectors. As the length of the panel increases, the difference in thermal bowing between fully and partially composite panels becomes negligible. Analysis of a long panel (example 1) with $L/r = 125$ shows that a relatively weak connection between wythes for a 10-m panel results in 82% of the thermal bowing experienced by a fully composite panel. This indicates that even noncomposite panels (which have some connecting layer stiffness) may experience problems with thermal bowing as the length-to-thickness ratio of the panel increases.

Comparing the displacement between the continuum model and finite element analysis shows that the accuracy of (15)–(20) is adequate for panel design. Analysis of a long panel shows a difference between finite-element and analytical results of 0.5%, with the analytical results underestimating the displacement. A similar analysis of a short panel shows an accuracy of 1%, with the analytical results overestimating the displacement.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A_c = cross-sectional area of connectors;
b = panel width;
d = structural wythe thickness;
E = modulus of elasticity of structural wythes;
E_c = modulus of elasticity of connectors;
e = connector elongation;
F = connector force;
I = panel moment of inertia;
I_c = connector moment of inertia;
I_w = structural wythe moment of inertia;
K = shear stiffness of connecting layer;
L = panel length;
M = total applied panel moment;
M_T = equivalent end moment caused by change in temperature ΔT;
m = number of connectors across panel width;
n = modular ratio E_c/E;
p = truss-type connector slope;
q = shear displacement between structural wythes;
r = distance between structural wythe centroids;
x = distance along panel axis;
y = lateral panel deflection;
α = constant: $\alpha^2 = (I - 2I_w)/I$;
α_T = coefficient of thermal expansion for structural wythe;
β = constant: $\beta^2 = 1 - \alpha^2$;
ΔT = relative temperature difference between two structural wythes;
δ = center panel deflection;
δ_0 = fully composite center panel deflection;
χ = constant: $\chi^2 = 2K/Ed$; and
ψ = $\chi L/2\beta$.